SCORE: _____ / 30 POINTS

1. NO CALCULATORS ALLOWED
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

Determine if the sequence $a_n = n\sin\frac{1}{n}$ converges or diverges. If it converges, find its limit. SCORE: 5 / 5 PTS
Justify your answer using proper algebra and/or calculus. State your conclusion clearly.

$f(x) = x\sin\frac{1}{x}$

$f'(x) = \sin\frac{1}{x} + x(\cos\frac{1}{x})\frac{-1}{x^2}$

$f'(x) = \sin\frac{1}{x} + \frac{\cos\frac{1}{x}}{} > 0$ for $x \geq 1$

increasing

$-1 \leq \sin\frac{1}{n} \leq 1$

$-n \leq n\sin\frac{1}{n} \leq n$

$\lim_{n\to\infty} n\sin\frac{1}{n}$

$\lim_{n\to\infty} \frac{\sin\frac{1}{n}}{1/n} \quad \frac{0}{0}$ 1½

L'Hop

$\lim_{n\to\infty} \frac{\cos\frac{1}{n} \cdot \cancel{\frac{-1}{n^2}}}{\cancel{\frac{-1}{n^2}}} \to \lim_{n\to\infty} \cos\frac{1}{n} = \boxed{1}$ 1½ 1½

converges because it has a limit ½

$n\to\infty$

Determine if $\left\{\frac{n^2}{\sqrt{n^3+4}}\right\}$ converges or diverges. If it converges, find its limit. SCORE: 0 / 4 PTS
Justify your answer using proper algebra and/or calculus. State your conclusion clearly.

$\lim_{n\to\infty} \frac{n^2}{\sqrt{n^3+4}} \cdot \frac{\sqrt{n^3+4}}{\sqrt{n^3+4}}$

$\lim_{n\to\infty} \frac{n^2\sqrt{n^3+4}}{n^3+4} \cdot \frac{1/n^3}{1/n^3}$

$\lim_{n\to\infty} \frac{\sqrt{n^7+4} \cdot 1/n^3}{1 + 4/n^3}$

$\lim_{n\to\infty} \frac{\sqrt{n + 4/n^6}}{1 + 4/n^3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$

$\lim_{n\to\infty} \frac{\sqrt{1 + 4/n^7}}{\frac{1}{\sqrt{n}} + 4/n^{3/2}} \quad \frac{\sqrt{1+\frac{4}{\infty}}}{\frac{1}{\infty} + 4/\infty}$

$= \boxed{1}$

Consider the sequence defined recursively by $a_1 = 3$, $a_{n+1} = \frac{a_n}{1+a_n}$. $\frac{3}{1+3}$, $\left(\frac{3}{4} \cdot \frac{1}{\frac{7}{4}}\right)$, $\frac{3}{7}$ · $\frac{1}{\frac{10}{7}}$ SCORE: 3 / 3 PTS

[a] List the first 4 terms of the sequence. Write your final answer as a list.

$\boxed{\left\{3, \frac{3}{4}, \frac{3}{7}, \frac{3}{10}, \ldots\right\}}$

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[b] Find a formula for the general term a_n of the sequence, assuming that the pattern of the first four terms from [a] continues.

$\boxed{a_n = \frac{3}{-2+3n}}$ 1

Determine if the sequence $a_n = \frac{\cos^2 n}{2^n}$ converges or diverges. If it converges, find its limit. SCORE: ____ / 4 PTS

Justify your answer using proper algebra and/or calculus. State your conclusion clearly.

$-1 < \cos^2 n < 1$ *

* $\frac{-1}{2^n} < \frac{\cos^2 n}{2^n} < \frac{1}{2^n}$

* $\lim_{n\to\infty} \frac{-1}{2^n} = 0$ $\lim_{n\to\infty} \frac{1}{2^n} = 0$

$\frac{1}{2}$

$\boxed{\text{so } \lim_{n\to\infty} \frac{\cos^2 n}{2^n} = 0}$ $\frac{1}{2}$ $\boxed{\text{converges}}$ because there is a limit

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Determine if the sequence defined recursively by $a_1 = 2$, $a_{n+1} = \frac{1}{2}(6 + a_n)$ is increasing or decreasing. SCORE: 4 / 4 PTS

Use mathematical induction to prove your answer. State your conclusion clearly.

$a_1 = 2$ $\{2, 4, 5 \ldots\}$

$a_2 = 4$

$\frac{1}{2}$ $a_2 - a_1 > 0$

so $\frac{1}{2}$ $a_{n+1} - a_n > 0$

$a_{n+1} > a_n$

$(6 + a_{n+1}) > (6 + a_n)$

$\frac{1}{2}(6 + a_{n+1}) > \frac{1}{2}(6 + a_n)$ 1

$a_{n+2} > a_{n+1}$ 1

Thus the sequence is $\boxed{\text{Increasing}}$

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